Contents

Welcome

It has been 15 years, almost to the day (well, all right, 15 years and one month) since the first **A**lgebra and **Sub**structural **L**ogics workshop ($\mathbf{A}_{\mathcal{L}}$, pronounced "A sub L"). Since then, the field has developed in several directions, taking hints and influences from areas as diverse as lattice-ordered group theory on the one hand, and Gentzen-style proof theory on the other. And has paid back in kind: witness the revival in ℓ-group theory brought about by the MV-algebra connection, the creation of algebraic proof theory, or the program of applying logical methods to general algebra.

The present $\mathbf{A}_{\mathcal{L}}$ being held where it is, one should expect two more themes to have a naturally relevant presence: relevant logics, and natural dualities.

I hope that this, fifth, edition of $\mathbf{A}_{\mathcal{L}}$ will prove as fruitful as the previous ones.

The local disorganiser
Tomasz Kowalski

Thanks

- To Australian Research Council grant FT100100952, for funding.
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Sunday, 14 December

- Wildlife Reserve excursion 11:00 – 12:30
- Catered BBQ lunch 12:30 – 15:00
- Informal discussions 15:00 – late

Monday, 15 December

- Registration 9:00 – 9:25

- Official opening 9:25 – 9:40

- Morning talks 9:40 – 12:40

09:40 – 10:20 Brian Davey (La Trobe University)
Surprise

10:20 – 11:00 Miroslav Haviar (M Bel University)
TiRS graphs and TiRS frames: a new setting for duals of canonical extensions

11:00 – 11:20 Tea/coffee break

11:20 – 12:00 Antonio Ledda (University of Cagliari)
Projectable ℓ-groups and algebras of logic: categorical and algebraic connections

12:00 – 12:40 Christopher Taylor (La Trobe University)
Discriminator varieties of Double-Heyting Algebras

- Afternoon talks 14:20 – 17:20

14:20 – 15:00 John Slaney (Australian National University)
What is Meyer's *single variable arrow fragment of E* problem?

15:00 – 15:40 Jan Kühr (Palacky University)
Residuated structures and certain triples

15:40 – 16:00 Tea/coffee break

16:00 – 16:40 Carles Noguera (Czech Academy Of Sciences)
Characterization of infinitary semilinear logics

16:40 – 17:20 Francesco Paoli (University of Cagliari)
Multiset consequence relation

Tuesday, 16 December

- Morning talks 9:00 – 12:40

09:00 – 09:40 Greg Restall (University of Melbourne)
Remarks on relevant arithmetic

09:40 – 10:20 Zach Weber (University of Otago)
Paraconsistent arithmetic without contraction

10:20 – 11:00 Martin Bunder (University of Wollongong)
Abstract versions of Gödel's Incompleteness and Tarski's Undefinability Theorems

11:00 – 11:20 Tea/coffee break

11:20 – 12:00 Petr Cintula (Czech Academy of Sciences)
Skolemization in substructural logics

12:00 – 12:40 Chris Fermueller (Technische Universität Wien)
From truth-functional to substructural logics via semantic games

- Afternoon talks 14:20 – 17:20

14:20 – 15:00 Hiroakira Ono (Japan Advanced Insitute of Science and Technology)
Uniform interpolation theorem for substructural predicate logics

15:00 – 15:40 Norbert Preining (Japan Advanced Insitute of Science and Technology)
Hyper-natural deduction system with normalization

15:40 – 16:00 Tea/coffee break

16:00 – 16:40 Marta Bilikova (Czech Academy of Sciences)
Substructural epistemic logic for skeptical agents

16:40 – 17:20 Tomasz Kowalski (La Trobe University)
Completions diagonalised away

Registrants

Marta Bilkova	Czech Academy of Sciences
Martin Bunder	University of Wollongong
Petr Cintula	Czech Academy of Sciences
Brian Davey	La Trobe University
Chris Fermüller	TU Vienna
Rohan French	Monash University
Asha Gair	La Trobe University
Charles Gray	La Trobe University
Miroslav Haviar	M Bel University, Slovakia
Katarzyna Idziak	Jagiellonian University
Jan Kühr	Palacky University Olomuc
Tomasz Kowalski	La Trobe University
Antonio Ledda	University of Cagliari
Carles Noguera	Czech Academy of Sciences
Hiroakira Ono	Japan Advanced Institute of Science and Technology
Francesco Paoli	University of Cagliari
Jane Pitkethly	La Trobe University
Norbert Preining	Japan Advanced Institute of Science and Technology
Greg Restall	The University of Melbourne
John Slaney	Australian National University
Christopher Taylor	La Trobe University
Zach Weber	University of Otago

Abstracts

Some abstracts are very concise, in the old logical tradition of allowing "zero or more" parts to constitute a whole.

Marta Bilkova (Institute of Computer Science, Czech Academy of Sciences)

Substructural epistemic logic for skeptical agents
16:00 Tuesday, 16 December

Martin Bunder (University of Wollongong)

Abstract versions of Gödel's incompleteness and Tarski's undefinability theorems
10:20 Tuesday, 16 December

with Joseph Tonien

This paper provides a condition for an arbitrary axiomatic system to be incomplete. We show that the condition applies to Peano Arithmetic as well as to the "puzzle" for self-describing machines of Smullyan and several well known paradoxes. We also prove a simple abstract version of Tarski's Undefinability Theorem and apply this to various set theoretic and semantic paradoxes.

Petr Cintula (Institute of Computer Science, Czech Academy of Sciences)

Skolemization in substructural logics
11:20 Tuesday, 16 December

Brian Davey (La Trobe University)

Surprise!
9:40 Monday, 15 December

Chris Fermüller (Technische Universität Wien)

From truth-functional to substructural logics via semantic games
12:00 Tuesday, 16 December

Game theoretic semantics is a versatile tool for characterizing classical and non-classical logics. We compare Hintikka-style and Giles-style semantic games for truth-functional many-valued logics. Although these two versions of game theoretic semantics seem to be quite different at a first glimpse, we show how a combination of the two paradigms leads to a natural characterization not only of many-valued connectives, but also of non-truth functional variants and extensions of those logics.

Miroslav Haviar (M Bel University, Slovakia)

TiRS graphs and TiRS frames: a new setting for duals of canonical extensions
10:20 Monday, 15 December

with Andrew P.K. Craig, Maria J. Gouveia

We consider properties of the graphs that arise as duals of bounded lattices in Ploscica's representation via maximal partial maps into the two-element set. We introduce TiRS graphs which abstract those duals of bounded lattices. We demonstrate their one-to-one correspondence with so-called TiRS frames which are a subclass of the class of RS frames introduced by Gehrke to represent perfect lattices. This yields a dual representation of finite lattices via finite TiRS frames, or equivalently finite TiRS graphs, which generalises the well-known Birkhoff dual representation of finite distributive lattices via finite posets. By using both Ploscica's and Gehrke's representations in tandem we present a new construction of the canonical extension of a bounded lattice. We present two open problems that will be of interest to researchers working in this area.

Jan Kühr (Palacky University Olomouc)

Residuated structures and certain triples
15:00 Monday, 15 December

Tomasz Kowalski (La Trobe University)

Completions diagonalised away
16:40 Tuesday, 16 December

A class of ordered algebras (typically, a quasivariety or a variety) is said to *admit completions* if every algebra in the class can be embedded into a complete one, also in the class. Failures of this property have long been known, and with some degree of poetic licence can be said to arise from two sources: modal algebras and ℓ-groups. These two sources have been thought to be rather disparate: at the very least, they seemed to require different techniques. We will show that they can be treated uniformly: as violations of a model-theoretic criterion, resembling diagonalisation to some extent.

Antonio Ledda (University of Cagliari)

Projectable ℓ-groups and algebras of logic: categorical and algebraic connections
11:20 Monday, 15 December

with Jose Gil-Ferez, Francesco Paoli, Constantine Tsinakis

In the 1960's, P.F. Conrad launched a general program for the investigation of lattice-ordered groups, aimed at capturing relevant information about these algebras by inquiring into the structure of their lattices of convex ℓ-subgroups (as opposed to convex *normal* ℓ-subgroups, which had traditionaly received greater attention in that they bijectively correspond to congruences). The chief idea behind this program is a working hypothesis to the effect that many significant properties of ℓ-groups are, in essence, either purely lattice-theoretical, or at least such that the underlying group structure does not play a predominant role. A class of ℓ-groups that is known to be characterized purely in terms of its order structure is the class of *projectable* ℓ-groups — namely, ℓ-groups in which every principal polar is a cardinal summand. One of the present authors, in fact, has established that an ℓ-group is projectable iff each one of its intervals is a Stone lattice; as a consequence, projectability is preserved under lattice isomorphisms. Also, the negative cone of an ℓ-group is projectable iff its lattice reduct can be endowed with a positive Gödel implication.

While Conrad's program led to remarkable outcomes in its original domain of application, a natural continuation of such consists in extending it to *residuated lattices*, generalizations of ℓ-groups that also include MV algebras, Heyting algebras, and several other classes of algebras of prime importance for mathematical logic. Here, the principal objects of research become the lattices of convex subalgebras (in the integral case, the lattices of multiplicative filters). Some detailed investigations along these lines have been carried out in recent years. One of the results obtained so far within this extended Conrad's program is a characterization of projectability for integral and distributive residuated lattices satisfying the quasiequation

$$x \vee y \approx 1 \Rightarrow xy \approx x \wedge y,$$

which closely matches the aforementioned description of projectable ℓ-groups. The last three authors of the present papers have indeed shown that a member of this class is projectable iff the order dual of each interval $[a, 1]$ is a Stone lattice.

In general, for integral and distributive residuated lattices, admitting a positive Gödel implication is a stronger condition than being projectable, although it is equivalent in some especially well-behaved cases. A case in point is given by *integral GMV algebras (IGMV algebras)*, simultaneous generalizations of MV algebras to the unbounded and noncommutative case. IGMV algebras, to within isomorphism, can be viewed as nucleus retractions of negative cones of ℓ-groups — actually, it has been shown that the categories of IGMV algebras and negative cones of ℓ-groups with a nucleus are equivalent. It is then natural to conjecture that such an equivalence restricts to an equivalence of the subcategories whose objects are the projectable members of these classes of algebras, and perhaps that we can take advantage of the previously cited lattice-theoretical description of projectable IGMV algebras to establish this result. The main aim of this talk is to investigate the extent to which this conjecture is correct.

Carles Noguera (Institute of Information Theory and Automation)

Characterization of infinitary semilinear logics
16:00 Monday, 15 December
with Petr Cintula

In our 2010 paper, Implicational (semilinear) logics I, we introduced a framework to study very general notions of implication and the classes of logics they determine. The crucial feature of our implications was that they induce an order relation in any general algebraic (matricial) model of the logic in question. One of the main results of the paper was a partial characterization (restricted to finitary logics) of the so-called semilinear implications and semilinear logics, i.e., logics complete w.r.t. the semantics in which the order induced by the implication is total. In this talk we present a full characterization by identifying the 'missing' property, which can be seen as generalized form of finitarity.

We also discuss three consequences of our work with an interest of their own. First we present a new hierarchy of logics, inspired by the above-mentioned property, characterized by the behavior of their (finitely) meet-irreducible filters/theories. Secondly, while proving the theorem we observed a flaw in the method of natural extensions (one of the important techniques of Abstract Algebraic Logic); here we give an example showing that the method really does not work in general but we also identify a very wide class of logics not affected by this problem. Finally we use the new characterization theorem to greatly strengthen several corollaries from the original paper.

Hiroakira Ono (Japan Advanced Institute of Science and Technology)

Uniform interpolation theorem for substructural predicate logics
14:20 Tuesday, 16 December

Francesco Paoli (University of Cagliari)

Multiset consequence relation
16:40 Monday, 15 December
with J. Gil Ferez, P. Cintula)

According to the dominant paradigm in Abstract Algebraic Logic (AAL), a (single-conclusion) consequence relation is a relation between a set of formulas and a formula of a given language. The theory of such consequence relations is well-rehearsed, and encompasses several different nonclassical logics, including substructural logics. It can be argued, however, that the official AAL notion of consequence relation does not do justice to some of the most subtle and intriguing features of these logics, for which a multiset-theoretical notion of consequence relation would seem more appropriate. Unfortunately, apart from a few pioneering studies e.g. by Avron, not much has been done on this count.

The aim of this talk is reporting on some preliminary investigations we carried out on multiset consequence relations. In particular:

- We introduce a multiple-conclusion notion of multiset consequence relation, as well as related concepts of multiset consequence operator and multiset closure system. We show that there is an order isomorphism between the complete lattices of multiset consequence operators, of multiset consequence relations and of multiset closure systems.
- We define and investigate multiset Hilbert systems.
- Finally, we introduce two forms of matrix semantics. As a case study, we prove a completeness theorem for the multiset-theoretical companion of infinite-valued Łukasiewicz logic.

Norbert Preining (Japan Advanced Institute of Science and Technology)

Hyper-natural deduction system with normalization
15:00 Tuesday, 16 December

We introduce a hyper natural deduction system that is sound and complete with respect to infinite-valued propositional Gödel Logic. We show translations to and from the hyper sequent calculus, as well as normalisation.

Greg Restall (The University of Melbourne)

Remarks on relevant arithmetic
9:00 Tuesday, 16 December

John Slaney (Australian National University)

What is Meyer's *single variable arrow fragment of E* problem?
14:20 Monday, 15 December

Christopher Taylor (La Trobe University)

Discriminator Varieties of Double-Heyting Algebras
12:00 Monday, 15 December

Just as Boolean algebras provide an algebraic counterpart to classical logic, Heyting algebras are the algebraic counterpart of intuitionistic logic, where the law of the excluded middle (i.e. "p or $\neg p$" is always true) is rejected. The rejection of this law means that implication $p \to q$ cannot be defined as $\neg p$ or q in an intuitionistic setting.

A Heyting algebra is a bounded distributive lattice endowed with an additional binary operation $\to$ (corresponding to intuitionistic implication). Including the dual of implication as a further binary operation gives us the class of *double-Heyting algebras.*

A *discriminator variety* is an equational class $\mathcal{V}$ that has a term $t(x, y, z)$ such that t is a *discriminator term* on every subdirectly irreducible member of $\mathcal{V}$, i.e. $t(x, y, z) = z$ if $x = y$ and x otherwise. Discriminator algebras are perhaps the most successful generalisation of Boolean algebras. It is well-known that any discriminator variety $\mathcal{V}$ is *semisimple*, that is, every subdirectly irreducible member of $\mathcal{V}$ is simple. However, the converse does not hold in general. We show that in the case of double-Heyting algebras, the converse is true.

Zach Weber (University of Otago)

Paraconsistent arithmetic without contraction
9:40 Tuesday, 16 December

Following a longstanding project in relevant logic, I will discuss some properties of an axiomatic substructural arithmetic. The main target, following unpublished work by Slaney, is to prove the irrationality of root 2, 'and no funny business'. Emphasis is placed on tricks and techniques for reasoning without the rule of contraction.